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**CALCULATION OF TWO-NEUTRON MULTIPLICITY  
IN PHOTONUCLEAR REACTIONS**

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**ABSTRACT**

The most important particle emission processes for electromagnetic excitations in nucleus-nucleus collisions are the ejection of single neutrons and protons and also pairs of neutrons and protons. Methods are presented for calculating two-neutron emission cross sections in photonuclear reactions. The results are in a form suitable for application to nucleus-nucleus reactions.

## **INTRODUCTION**

When cosmic rays in the form of heavy nuclei pass through spacecraft walls and astronauts' bodies, they undergo an interaction with the atomic nuclei in the spacecraft or astronauts. Of the four forces that we currently know about (strong, weak, electromagnetic and gravitational), the cosmic ray interaction occurs via both the strong and electromagnetic forces. The strong and electromagnetic (EM) cross sections are of comparable magnitude in some situations. Previous work (refs. 1-5) has concentrated on studying this electromagnetic aspect of nucleus-nucleus collisions. This study has so far considered only single-nucleon emission processes. A preliminary study has been made of multiple-nucleon emission (ref. 5). However, the results of this study are unable to be utilized in heavy-ion transport codes because experimental photonuclear cross sections are used as inputs into the calculation of nucleus-nucleus EM cross sections.

The aim of the present work is to show how to calculate these photonuclear cross sections for multiple-nucleon emission. Given analytic expressions for these cross sections, it will then be possible to add multiple-nucleon emission due to the EM effect into heavy-ion transport codes. Calculating multiple-nucleon emission effects for the EM interaction is much more difficult than the calculation for single-nucleon emission. Thus the present work will consider only the most important multiple-nucleon emission process—that of two-neutron emission. Other multiple-nucleon effects such as emission of two protons, a neutron and a proton, or an alpha particle are more strongly suppressed than two-neutron emission primarily due to the Coulomb barrier. This is especially true for heavy nuclei. In fact even single-proton emission is completely Coulomb-suppressed for heavy nuclei (ref. 1).

## MULTIPLE NUCLEON EMISSION AND ARBITRARY ELECTROMAGNETIC MULTIPOLES

Before proceeding to the study of two-neutron emission, we shall first place it in the broad context of arbitrary EM multipoles and multiple-nucleon emission.

The total nucleus-nucleus EM absorption cross section  $\sigma_{EM}$  is given by

$$\sigma_{EM} = \sum_x \sigma_{EM}^{(x)} \quad (1)$$

where the sum is over all possible species emitted in the heavy ion collision and  $\sigma_{EM}^{(x)}$  is the nucleus-nucleus EM reaction cross section for producing a particular species  $x$ . The cross section  $\sigma_{EM}^{(x)}$  was given as equation (1) in ref. 1, but we now generalize it as (see also equation 2.1 of ref. 6 and equation 4.11 of ref. 7)

$$\sigma_{EM}^{(x)} = \sum_{\pi' l} \int_{E_0(x)} \sigma_v^{\pi' l}(E, x) N^{\pi' l}(E) dE \quad (2)$$

where  $\pi$  can be either electric ( $\mathcal{E}$ ) or magnetic ( $\mathcal{M}$ ) and  $l$  is the order of multipolarity. Each term in the summation of equation (2) represents nuclear excitation by a particular EM multipole. Each different EM multipole  $\pi l$  causes a particular type of nuclear excitation. For example, the  $\mathcal{E}1$  photon field causes the nucleus to go into the giant electric dipole resonance (GDR) mode of oscillation parameterized as

$$\sigma_{abs}^{\mathcal{E}1}(E) = \frac{\sigma_m}{1 + [(E^2 - E_{GDR}^2)/E^2 T^2]}, \quad (3)$$

(see equation 8 of ref. 1 and equation 6 below) which would subsequently decay into various channels  $x$ , with a probability  $g_x^{\mathcal{E}1}$ , sometimes called the branching ratio.

However, an  $\mathcal{E}2$  multipole (ref. 8) would cause the nucleus to go into a giant electric quadrupole resonance (GQR), the cross section of which would have a different parameterization from equation (3), and where various decay probabilities  $g_x^{\mathcal{E}2}$  (e.g., neutron versus proton decay) may also be different from  $g_x^{\mathcal{E}1}$ . Thus the photonuclear

reaction cross section for production of species  $x$  is some fraction of the total photonuclear absorption cross section

$$\sigma_v^{\pi'}(E, x) = g_x^{\pi'}(E) \sigma_{\text{abs}}^{\pi'}(E) \quad (4)$$

for a particular multipole. Equation (4) is a generalization of equation (7) of ref. 1, and means physically that a particular EM multipole  $\pi'$ , causes a collective nuclear vibration  $\sigma_{\text{abs}}^{\pi'}(E)$  which can then decay into various channels via  $g_x^{\pi'}(E)$ . Note that

$$\sum_x g_x^{\pi'}(E) = 1 \quad (5)$$

By combining equations (2) and (4) and assuming an energy independent branching ratio (replacing  $g_x^{\pi'}(E)$  with  $g_x^{\pi'}$ ), the nucleus-nucleus EM reaction cross section becomes

$$\sigma_{\text{EM}}(x) \approx \sum_{\pi'} g_x^{\pi'} \int_{E_0(x)} \sigma_{\text{abs}}^{\pi'}(E) N^{\pi'}(E) dE = \sum_{\pi'} g_x^{\pi'} \sigma_{\text{EM-abs}}^{\pi'} \quad (6)$$

where the nucleus-nucleus EM total absorption cross section is

$$\sigma_{\text{EM-abs}}^{\pi'} \equiv \int_{E_0(x)} \sigma_{\text{abs}}^{\pi'}(E) N^{\pi'}(E) dE. \quad (7)$$

The above three equations are generalizations of the three equations on page 7 of ref. 1. (Note that in ref. 1 the absorption cross section was written as  $\sigma_{\text{EM-abs}}(x)$  in equation (15) of ref. 1. The dependence on  $x$  came out because the threshold  $E_0(x)$  is the lower limit of integration. We prefer here simply to write  $\sigma_{\text{EM-abs}}$  because a true absorption cross section should not depend on  $x$ .) In the above equations, it is the photonuclear total absorption cross section  $\sigma_{\text{abs}}^{\pi'}(E)$  which gets parameterized according to the particular nuclear multipole excitation. For example the nuclear  $\mathcal{E}1$  GDR excitation is parameterized in equation (3).

If one assumes that the branching ratios are independent of multipolarity (which may receive some justification from the Bohr independence hypothesis (ref. 9)), then, replacing  $g_x^{\pi'}$  with  $g_x$ , equation (6) becomes

$$\sigma_{EM}(E) \approx g_x \sum_{\pi l} \sigma_{EM-abs}^{\pi'} = g_x \sigma_{EM-abs} \quad (8)$$

where the nucleus-nucleus EM total absorption cross section summed over all EM multipoles is

$$\sigma_{EM-abs} = \sum_{\pi'} \sigma_{EM-abs}^{\pi'} \quad (9)$$

Equation (8) is interpreted physically as meaning that a nucleus-nucleus EM reaction has occurred exciting a nucleus into a superposition of multipolarities  $\sum_{\pi'} \sigma_{EM-abs}^{\pi'}$  (such as

a linear combination of GDR and GQR). This superposition can be likened to the Compound Nucleus concept of Blatt and Weisskopf (ref. 9). This compound nucleus superposition then decays via multipole-independent branching ratios  $g_x$  as in equation (8).

Although equation (6) is probably more correct, it will be impossible to implement in practice due to the difficulty of calculating multipole-dependent branching ratios. Therefore in practical calculations, equation (8) will be used. This equation is entirely consistent with the concept of compound nucleus formation (ref. 9) and decay independent of the mode of formation (Bohr independence hypothesis). This is identical in spirit to the Abrasion-Ablation model (ref. 10). In fact, attempts (ref. 11) have been made to understand the Abrasion-Ablation model in terms of compound nucleus formation and decay using a T-matrix approach. Thus our basic equation (8) can be thought of as an Electromagnetic Abrasion-Ablation model.

Finally, we put our previous studies (refs. 1-5) in the context of equations (1) and (8). First of all, equation (8) tells us that in order to calculate the nucleus-nucleus EM

reaction cross section, we should sum over all possible EM multipolarities ( $\mathcal{E}0, \mathcal{E}1, \mathcal{E}2 \dots, \mathcal{M}0, \mathcal{M}1, \mathcal{M}2 \dots$ ). The only multipolarity that has been studied so far is  $\mathcal{E}1$  (ref. 1-5) leading to the Giant Electric Dipole Resonance. Clearly the effects of other multipolarities must be considered. Bertulani and Baur have concluded (ref. 6) that the electric quadrupole ( $\mathcal{E}2$ ) contribution to the total nucleus-nucleus EM cross section can be as much as 50 percent of the  $\mathcal{E}1$  contribution at 100 MeV/N and about 20 percent at 1 GeV/N dropping to about 10 percent at higher energies. Note that all the EM data (see ref. 1) are at high energy and so in comparing our theory (ref. 1) with experiment, the  $\mathcal{E}2$  contribution has not been large. Nevertheless we require the nucleus-nucleus (fragmentation and EM) theory to include all energies in the cosmic ray spectrum, and thus it is very important to consider other EM multipoles as well. Apart from multipoles other than  $\mathcal{E}1$ , equation (1) tells us that we need also consider not only single-nucleon emission but multiple-nucleon emission as well, such as emission of np, 2n, 2p,  $\alpha$ , nnp, npp and 3n. However, references 1-4 have considered only the  $\mathcal{E}1$  multipole and only single-nucleon emission. Thus there still remains much territory to explore, namely multiple-nucleon emission with  $\mathcal{E}1$  excitation and then single-nucleon and multiple-nucleon emission for all other multipoles. The present paper is concerned only with two-neutron (2n) emission from  $\mathcal{E}1$  excitation.

## THEORY OF TWO-NEUTRON EMISSION

Cucinotta et al. (ref. 5) have in fact studied the problem of multiple-nucleon emission ( $2n$ ,  $2p$ ,  $p3n$ ,  $3n$ ) with  $\mathcal{E}1$  excitations only. However, the photonuclear reaction cross section  $\sigma_v(E, x)$  was obtained from experimental data, and so even though useful conclusions were drawn, the work of ref. 5 was incomplete and cannot be used in a transport code, which requires analytical expressions for  $\sigma_v^{\pi'}(E, x)$ . The aim of the present work therefore is to begin the study of multiple nucleon emission in nucleus-nucleus electric dipole ( $\mathcal{E}1$ ) excitation reactions using an analytic approach. The motivation for such an approach is to implement analytic cross section expressions into cosmic ray transport codes. There is no cosmic ray transport code in existence which includes anything other than single-nucleon emission for EM reactions. Because a full theory for multiple-nucleon emission is much more complicated than for single-nucleon emission, the present work will be limited to a study of two-neutron emission, which is the dominant multiple-nucleon contribution.

## TWO-NEUTRON MULTIPLICITY

Fuller et al. (ref. 12, pp. 190; ref. 13, pp. 4; ref. 14, pp. 143-145) have defined the total photoneutron yield cross section as

$$\sigma(\gamma, xn) \equiv \sigma(\gamma, n) + 2\sigma(\gamma, 2n) + 3\sigma(\gamma, 3n) + \dots \quad (10)$$

and the photoneutron cross section (i.e., the sum of cross sections in which at least one neutron is emitted)

$$\sigma(\gamma, sn) \equiv \sigma(\gamma, n) + \sigma(\gamma, np) + \sigma(\gamma, 2n) + \sigma(\gamma, n\alpha) + \sigma(\gamma, 3n) + \dots \quad (11)$$

The neutron multiplicity is defined (ref. 15) as

$$M(E) \equiv \frac{\sigma(\gamma, xn)}{\sigma(\gamma, sn)} \quad (12)$$

which for  $n$  and  $2n$  emission only becomes (ref. 16)

$$M(E) = \frac{\sigma(\gamma, n) + 2\sigma(\gamma, 2n)}{\sigma(\gamma, n) + \sigma(\gamma, 2n)} \quad (13)$$

This expression for the multiplicity has been implicitly assumed also in Thies and Spicer (ref. 17, equation 5). Note the reason for writing the neutron multiplicity this way: for  $\sigma(\gamma, 2n)$  equal to zero (meaning that only single neutrons ( $n$ ) are emitted), the multiplicity is 1 as expected, and for  $\sigma(\gamma, n)$  being zero (meaning only two-neutron pairs ( $2n$ ) are emitted), the multiplicity is 2 as expected. Often instead of using multiplicity, one works with the quantity

$$\frac{\sigma(\gamma, 2n)}{\sigma(\gamma, n) + \sigma(\gamma, 2n)} = M(E) - 1 \quad (14)$$

Blatt and Weisskopf (ref. 9, chapter VIII, section 6B) have worked out the general theory of multiple-nucleon emission, which they call secondary nuclear reactions. They write the cross section for production of particle  $b$  (see ref. 9, equation on pp. 375), which we specialize to photoproduction as

$$\sigma(\gamma, sb) = \sigma(\gamma, b) + \sum_c \sigma(\gamma, bc) \quad (15)$$

This is made up of a primary, single-step cross section  $\sigma(\gamma, b)$  in which particle  $b$  is emitted directly from decay of the compound nucleus and a secondary, multiple-step cross section  $\sigma(\gamma, bc)$  in which either particle  $b$  or  $c$  is emitted from the compound nucleus which decays to a lower excited state which again decays via emission of particle  $c$  or  $b$ . Blatt and Weisskopf (ref. 9, pp. 376-379) then go on to calculate  $\sigma(\gamma, b)$  and  $\sigma(\gamma, bc)$  in terms of  $\sigma(\gamma, sb)$ ; however, one generally needs to work the integrals out numerically. We shall eventually do this when working out the general expression for multiple-nucleon emission. However, for the case of two-neutron emission, certain simplifying assumptions concerning the integrals can be made so that analytic expressions can be obtained. These are (ref. 9, eqn. 6.14, pp. 377)

$$\sigma(\gamma, n) \approx \sigma(\gamma, sn) (1 + \epsilon_{sec}/\Theta) \exp(-\epsilon_{sec}/\Theta) \quad (16)$$



and (ref. 9, eqn 6.18, pp.379)

$$\sigma(\gamma, 2n) \approx \sigma(\gamma, sn) [1 - (1 + \epsilon_{sec}/\Theta)\exp(-\epsilon_{sec}/\Theta)] \quad (17)$$

where  $\Theta$  is the nuclear temperature and

$$\epsilon_{sec} \equiv E\gamma - E_o(\gamma, 2n) \quad (18)$$

The above equations are also discussed in references 15, 16, 17, 18. Equation (17) is derived on the assumption that the photoneutrons are produced via statistical decay of the compound nucleus (ref. 9). However this is not always necessarily true. It can sometimes happen that the incident photon, rather than exciting a compound nucleus which subsequently decays via neutron emission, will rather knock a neutron out directly. (Note that this direct emission violates the abrasion-ablation concept.) We therefore introduce a fraction of direct emission  $f_d$ , which typically has values (ref. 15, and G. O'Keefe and R. Rassool, private communication) in the range 0.1 - 0.2 although it can be as large as 0.4 (ref. 17). This direct fraction is incorporated into our theory (ref. 15, 17) by generalizing equation (17) as

$$\sigma(\gamma, 2n) \approx \sigma(\gamma, sn) (1 - f_d) [1 - (1 + \epsilon_{sec}/\Theta) \exp(-\epsilon_{sec}/\Theta)] \quad (19)$$

This equation is our main result for 2n emission. The quantities  $f_d$ ,  $\epsilon_{sec}$ , and  $\Theta$  are easily calculated so that the only input required is  $\sigma(\gamma, sn)$ . Blatt and Weisskopf (ref. 9, pp. 379) claim that  $\sigma(\gamma, sn)$  can usually be closely approximated by the compound nucleus formation cross section, i.e., the total absorption cross section. (This is done in reference 18.)

However, this is only true when compound nucleus decay proceeds predominantly by neutron emission which is the case for heavy nuclei. For light nuclei, where proton emission is just as important (ref. 1), the above approximation is not valid. Thus we now neglect all contributions to  $\sigma(\gamma, sn)$  other than  $\sigma(\gamma, n)$  and  $\sigma(\gamma, 2n)$  so that

$$\frac{\sigma(\gamma, 2n)}{\sigma(\gamma, n) + \sigma(\gamma, 2n)} \approx (1 - f_d) [1 - (1 + \epsilon_{sec}/\Theta) \exp(-\epsilon_{sec}/\Theta)] \quad (20)$$

which is the result written in reference 16 and 18 for  $f_d = 0$ . Defining

$$X_{frac} \equiv (1 - f_d) [1 - (1 + \epsilon_{sec}/\Theta) \exp(-\epsilon_{sec}/\Theta)] \quad (21)$$

we rearrange (20) to obtain

$$\sigma(\gamma, 2n) = \frac{X_{\text{frac}}}{1 - X_{\text{frac}}} \sigma(\gamma, n). \quad (22)$$

Although equation (19) is our fundamental equation for 2n emission, we shall in fact use equation (22) to test the general theory. This is because several authors have reported both  $\sigma(\gamma, n)$  and  $\sigma(\gamma, 2n)$ . We shall use experimental input for  $\sigma(\gamma, n)$  and then calculate  $\sigma(\gamma, 2n)$  using (22) and see how well our calculations agree with experimental measurements. If we obtain good results, we can be confident that the theory outlined above can be used to calculate 2n emission in EM nucleus-nucleus collisions.

Finally, we note that the above equations can be used for an approximate calculation of the neutron multiplicity given in equation (13) as

$$M(E) = 1 + (1 - f_d)[1 - (1 + \epsilon_{\text{sec}}/\Theta)\exp(-\epsilon_{\text{sec}}/\Theta)] \quad (23)$$

which is the result reported in references 15 and 17. (Note however the error in equation (3.12) of reference 15, which is that  $\sigma(\gamma, np)$  should not appear in the numerator).

## NUCLEAR TEMPERATURE

In the above expressions, the nuclear temperature  $\Theta$  plays a central role. For a Maxwell-Boltzmann (classical) gas (ref. 19, pp. 117), the relation between energy and temperature is linear

$$E = \frac{3}{2} N_0 k \Theta \quad (24)$$

whereas for a Fermi gas, the relation is quadratic

$$E = (\text{const.}) k^2 \Theta^2 \quad (25)$$

The theory developed by Blatt and Weisskopf (ref. 9, pp. 372) uses a Fermi gas nuclear temperature given by

$$\Theta = \left( \frac{E}{\mathcal{A}} \right)^{1/2} \quad (26)$$

where the constant  $\mathcal{A}$  depends on the total nucleon number  $A$ . We shall not use the Blatt and Weisskopf theory for nuclear temperature, but rather use a more sophisticated version developed by Bohr and Mottelson where (ref. 20, eqn. 2-50, pp. 154)

$$\Theta^{-1} = \frac{1}{\rho} \frac{\partial \rho}{\partial E} \quad (27)$$

with the Fermi gas energy level density (ref. 20, eqn. 2-47, pp. 153)

$$\rho(N, Z, E) = \frac{6^{1/4}}{12} \frac{g_0}{(g_0 E)^{5/4}} \exp \left[ 2 \left( \frac{\pi^2}{6} g_0 E \right)^{1/2} \right] \quad N \approx Z \quad (28)$$

being of similar form to that of Blatt and Weisskopf (ref. 9, eqn. 2-50, pp. 154). The symbol  $E$  is the excitation energy, and  $g_0$  is the "one-particle level density at the Fermi energy, representing the sum of neutron and proton level densities" (ref. 20, pp. 153).

Thus the nuclear temperature becomes (ref. 20, eqn. 2-50, pp. 154)

$$\Theta^{-1} = \left( \frac{\pi^2}{6} \frac{g_0}{E} \right)^{1/2} - \frac{5}{4E} \quad (29)$$

which is the same form as used in reference 15. It is very important to realize that the energy  $E$  in the above expressions is, in fact

$$E = E_\gamma - E_0(\gamma, n) \quad (30)$$

given by the single-neutron emission threshold  $E_0(\gamma, n)$ . This is emphasized in references 18 (pp.304) and by G. O'Keefe and R. Rassool (private communication). The one-particle level density  $g_0$  is related to the level density parameter  $a$  via (ref. 20, eqn. 2-123, pp. 187)

$$a = \frac{\pi^2}{6} g_0, \quad (31)$$

which is plotted in Figures 2-12 of reference 20 (pp. 187). It is seen that  $a$  very much depends on shell structure, and therefore whenever possible,  $a$  should be obtained directly from Figures 2-12 (ref. 20). However, a rough approximation is

$$a \approx \frac{A}{8} (\text{MeV}^{-1}) \quad (32)$$

but note that this fails badly for  $A$  between 190 and 210, (ref. 20).

Rewriting the nuclear temperature as

$$\Theta^{-1} = \left( \frac{a}{E} \right)^{1/2} - \frac{5}{4E} \quad (33)$$

with the first term corresponding to the form used by Blatt and Weisskopf (ref. 9), we approximate  $a$  to obtain

$$\Theta^{-1} = \left( \frac{A}{8E} \right)^{1/2} - \frac{5}{4E} \quad (34).$$

## **TEST OF TWO-NEUTRON MULTIPLICITY THEORY**

A computer code was written to implement the theory described above. Necessary inputs are the energy level density parameter  $a$  which was taken directly from Figs. 2-12 (ref. 20, pp. 187) rather than from equation (32). The two-neutron cross section was then calculated from the single-neutron cross section with a best-fit value of  $f_d$ , the fraction of direct emission. The results are listed in Tables 1-8. As can be seen, the agreement between theory and experiment is good, except for a few notable exceptions. It is disturbing, however, that the direct emission fractions are so high.

## **DISCUSSION**

We have demonstrated that the theory of two-neutron emission described herein is able to predict most two-neutron multiplicities with a high degree of accuracy. One therefore wishes to implement this theory into the calculation of nucleus-nucleus cross sections and eventually into a transport code. A parameterization of the energy level density parameter  $a$  can be very easily worked out (ref. 20). Thus the only unknowns are  $f_d$  and  $\sigma(\gamma, n)$ .

The values of  $f_d$  were obtained from an overall best fit to  $\sigma(\gamma, 2n)$  data. (In practice, a single  $\sigma(\gamma, 2n)$  value at a single energy determines  $f_d$ .) Clearly some new work remains to be done here. One can either work out a parameterization by determining  $f_d$  from best fits to a whole range of data, or better still, one should determine  $f_d$  theoretically. Our initial effort will undoubtedly be a parameterization, so that the theory can be used within transport codes.

The most serious unknown is  $\sigma(\gamma, n)$  which is used as input for the  $\sigma(\gamma, 2n)$  calculation at each energy. One might think that the liquid drop model calculations for  $\sigma(\gamma, n)$ , described extensively in reference 1, would be adequate for this purpose, especially as these same model calculations gave such good results for nucleus-nucleus cross sections. Unfortunately, this is not the case. Even though the liquid drop model calculations of  $\sigma(\gamma, n)$  are quite accurate at most photon energies, they are unfortunately not very accurate for high photon energies. The reason that this did not matter for the calculations of reference 1 was that the virtual photon spectrum  $N(E)$  predominated at low photon energies, and so energy integrated nucleus-nucleus cross sections were quite accurate. However, obviously  $(\gamma, 2n)$  processes only occur in the high energy  $(\gamma, n)$  region. The  $\sigma(\gamma, 2n)$  calculations require  $\sigma(\gamma, n)$  as input, and thus we now need very accurate calculations for  $\sigma(\gamma, n)$  at high energy. (Calculations described in the present paper used experimental numbers.)

Thus, the most pressing need for inclusion of the effects of multiple-nucleon emission in heavy ion transport codes is the accurate calculation of  $\sigma(\gamma, n)$  at high energy. The first such step will require the study of deformation splitting (ref. 14, 17, 18, 24, 25) and isospin splitting of the giant dipole resonance (ref. 24, 25).

Finally, even though only two-neutron emission has been considered herein, it is now clear how to proceed with the general multiple-nucleon emission problem. One can simply numerically integrate the expressions of Blatt and Weisskopf (ref. 9) to obtain any multiple-nucleon final state. (These expressions simplified in the two-neutron case considered herein, so that numerical integration was not necessary.) This method has previously been applied to  $^{32}\text{S}(\gamma, d)$  by Norbury (ref. 26).

As an alternative to the whole approach taken in the present work (and even for the single-nucleon work of ref. 1), it may be feasible to instead do calculations using some of the modern nuclear evaporation codes (e.g., EVAP-4 and EVA-3). The advantage of these codes is that they fully incorporate both statistical and direct emission. The excitation energy in these codes is simply the photon energy considered above.

## **CONCLUSIONS**

Electromagnetic excitations in nucleus-nucleus collisions can often involve the ejection of pairs of nucleons. Methods have been presented for calculating two-neutron emission cross sections in photonuclear reactions. These cross sections are now in a form suitable for use in nucleus-nucleus reaction theory.

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**Table 1**  $^{12}\text{C}$  photoneutron cross sections. Data are from ref. 21

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.9$ $a = 2 \text{ MeV}^{-1}$
35	2	0.004	0.043
40	0.8	0.012	0.051
50	0.2	0.015	0.020
60	0.1	0.014	0.011
80	0.1	0.095	0.011

**Table 2**  $^{18}\text{O}$  photoneutron cross sections. Data are from ref. 27

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.5$ $a = 3 \text{ MeV}^{-1}$
14	4	1	0.5
18	2.4	3	1.3
22	4	3	3.1
26	7	3	6
30	5	3	5
34	2	3	1.9
38	2	2	1.9
42	1	3	1.2

**Table 3**  $^{40}\text{Ar}$  photoneutron cross sections. Data are from ref. 22

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.05$ $a = 6 \text{ MeV}^{-1}$
18	16	5	6
20	7	14	12
21	5	13	14
22	3	11	12
23	2	10	11

**Table 4**  $^{59}\text{Co}$  photoneutron cross sections. Data are from ref. 28

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.6$ $a = 8 \text{ MeV}^{-1}$
18	65	0.3	27
20	47	9	4
22	21	17	7
24	17	12	9
26	14	11	8
28	19	9	12
30	20	5	13
32	9	6	6
34	1	6	1
36	4	3	3

**Table 5**  $^{63}\text{Cu}$  photoneutron cross sections. Data are from ref. 23

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.3$ $a = 8 \text{ MeV}^{-1}$
20	44	2	0.6
21	26	4	5
22	20	8	11
23	13	12	12
24	11	13	13
25	10	13	15

**Table 6**  $^{89}\text{Y}$  photoneutron cross sections. Data are from ref. 29

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.5$ $a = 12 \text{ MeV}^{-1}$
20	70	0	36
22	35	7	6
24	10	10	6
26	2	16	2
28	0.5	16	0.5

**Table 7**  $^{197}\text{Au}$  photoneutron cross sections. Data are from ref. 16 but have been multiplied by 0.93 following the suggestion of ref. 30

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.4$ $a = 24 \text{ MeV}^{-1}$
15	391	9	20
16	214	73	121
17	93	98	98
18	56	93	74
19	37	65	53
20	27	58	40
21	17	42	25
22	11	47	16
23	7	40	11
24	2	37	3



**Table 8**  $^{208}\text{Pb}$  photoneutron cross sections. Data are from ref. 16

$E_\gamma$ , MeV	$\sigma(\gamma, n)$ , mb	$\sigma(\gamma, 2n)$ , mb	
		Experimental	Theoretical $f_d = 0.2$ $a = 8 \text{ MeV}^{-1}$
16	180	80	104
18	30	80	51
20	20	50	55

## LIST OF SYMBOLS

$A$	nuclear temperature constant, MeV <sup>-1</sup>
$a$	energy level density parameter, MeV <sup>-1</sup>
$E$	energy, MeV
EM	abbreviation for "electromagnetic"
$E_{\text{GDR}}$	giant dipole resonance central energy, MeV
$E_0(x)$	photonuclear reaction threshold for a particular species $x$ , MeV
$E_\gamma$	photon energy, MeV
$\ell$	electric multipole of multipolarity $\ell$
$f_d$	fraction of direct, non-statistical emission
$g_0$	one particle level density at the Fermi energy, MeV <sup>-1</sup>
$g_x^{\pi'}$	energy independent branching ratio for species $x$ due to a $\pi'$ nuclear excitation
$g_x^{\pi'}(E)$	energy dependent branching ratio for species $x$ due to a $\pi'$ nuclear excitation
$k$	Boltzmann constant $1.3805 \times 10^{-23}$ J/K
$l$	multipolarity
$M\ell$	magnetic multipole of multipolarity $\ell$
mb	millibarn
$N_0$	number of particles
$N^{\pi'}(E)$	virtual photon number spectrum of a particular multipole, MeV <sup>-1</sup>
$n$	neutron
$p$	proton
$x$	particular particle species emitted in a heavy ion collision
$\alpha$	alpha particle
$\Gamma$	giant dipole resonance width, MeV
$\epsilon_{\text{sec}}$	maximum energy available for emission of secondary particles, MeV

$\pi /$	particular electromagnetic multipole (e.g., $E1, M0$ )
$\rho$	energy level density, $\text{MeV}^{-1}$
$\sigma$	cross section, mb
$\sigma(\gamma, sb)$	sum of cross sections in which at least one particle b is emitted
$\sigma(\gamma, sn)$	photoneutron cross section, mb
$\sigma(\gamma, xn)$	total photoneutron yield cross section, mb
$\sigma_{\text{abs}}^{\pi /}(E)$	photonuclear total absorption cross section, mb
$\sigma_{\text{EM}}$	total nucleus-nucleus EM absorption cross section, mb
$\sigma_{\text{EM}}(x)$	nucleus-nucleus EM reaction cross section for production of a particular species x, mb
$\sigma_{\text{EM-abs}}$	nucleus-nucleus EM total absorption cross section summed over all EM multipoles, mb
$\sigma_{\text{EM-abs}}^{\pi /}$	nucleus-nucleus EM total absorption cross section for a particular multipole, mb
$\sigma_m$	giant dipole resonance cross section peak value, mb
$\sigma_v^{\pi /}(E, x)$	photonuclear reaction cross section for production of a particular species x due to nuclear excitation from a particular multipole $\pi /$ , mb